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Nuclear Physics B 896 (2015) 1–18

NUCLEAR PHYSICS B

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Lagrangian formulation of massive fermionic higher spin fields on a constant electromagnetic background

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Received 21 January 2015; received in revised form 10 March 2015; accepted 10 April 2015

Available online 14 April 2015

Editor: Stephan Stieberger

Abstract

We consider massive half-integer higher spin fields coupled to an external constant electromagnetic field in flat space of an arbitrary dimension and construct a gauge invariant Lagrangian in the linear approximation in the external field. A procedure for finding the gauge-invariant Lagrangians is based on the BRST construction where no off-shell constraints on the fields and on the gauge parameters are imposed from the very beginning. As an example of the general procedure, we derive a gauge invariant Lagrangian for a massive fermionic field with spin 3/2 which contains a set of auxiliary fields and gauge symmetries.

1. Introduction

Despite the recent progress in higher spin gauge theories (see e.g. [1–16] for review of various aspects of the subject) there are still a number of problems to address. Construction of the interacting Lagrangians of massive higher spin fields on various backgrounds and study of the properties of these systems is one of these problems. Apart from being interesting in its own right,

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http://dx.doi.org/10.1016/j.nuclphysb.2015.04.008

it is also important from the string theory perspective [17]. As is well known, string theory contains an infinite tower of massive higher spin modes and therefore it is important to understand on which backgrounds these fields can propagate consistently.

Although many aspects of Lagrangian formulation of free fermionic higher spin fields have been studied well enough (see e.g. [18–20] and the references therein) the problem of interacting fermionic fields is much less understood than the problem of interacting bosonic fields (see also [16] for a recent review). In particular, that the cubic vertices which include fermionic higher spin fields have been constructed in the light cone framework in [21] and various problems of interaction with gravitational and electromagnetic fields have been addressed in [22–34].[1]

When considering interactions of massive fields with spin more than zero with a nontrivial background one faces several difficulties such as superluminal propagation and violation of the number of physical degrees of freedom. The requirement that no superluminal propagation takes place imposes in general certain conditions on the background fields [40,41] (see also [42] for a recent discussion). Similarly, when turning on nonzero background fields the invariance of the initial system under its gauge transformations can be partially or completely lost and this means in turn that nonphysical polarizations can appear in the spectrum. The requirement of preserving of physical degrees of freedom generically imposes some extra conditions on the background. The question is therefore to find if a background under consideration is physically acceptable i.e., if it satisfies the constraints imposed by the above mentioned conditions.

In this paper we consider a problem of interaction of massive totally symmetric fermionic higher spin fields with constant electromagnetic (EM) background in Minkowski space of an arbitrary dimension d. These higher spin fields are described by tensor–spinors with one spinorial index and an arbitrary number $n = s - 1/2$ of totally symmetric tensorial indices. Our main aim is to derive the gauge invariant Lagrangian using the method of BRST construction in the linear approximation in strength $F_{\mu\nu}$ of the external field. This method in fact yields a gauge invariant Lagrangian description for massive higher spin fields in extended Fock space and therefore the Lagrangian will contain, apart from the basic fields, some extra auxiliary fields such as Stückelberg fields. Some of these fields are eliminated with the help of gauge transformations, some of the others should be eliminated as a result of the equations of motion. Therefore, in order to have a consistent gauge invariant description for massive higher spin fields, one should have enough gauge freedom and have the "correct" equations of motion, which ensure the absence of ghosts.[2] Performing this analysis in a way similar to how it has been done in [43] one can show that the preservation of physical degrees of freedom indeed takes place for the Lagrangian under consideration, provided that the terms containing the strength of the external space are considered as a perturbation. Where the problem of superluminal propagation of higher spin fields is concerned we note that in the linear in $F_{\mu\nu}$ approximation this problem does not arise at all due to antisymmetry of $F_{\mu\nu}$ (see e.g. [40] for a spin 3/2 field).

The paper is organized as follows. Section 2 contains our main results. After a brief reminder of construction of Lagrangians for free massive fermionic higher spin fields we introduce interaction with background electromagnetic fields by modifying the operators which define the BRST charge. The requirement that the modified operators form a closed algebra determines free pa-

[1] Also one points out the papers [36,37] where non-Lagrangian equations of motion for higher spin fields in the external fields have been considered.

[2] One way to check this is to perform a complete gauge fixing in the equations of motion and obtain the equations in terms of basic fields. As a result one obtains equations defining the spectrum of the theory and check if it is ghost free or not.

rameters which are present in the definition of the operators. Then we present the corresponding BRST charge, construct the Lagrangian and use a part of the BRST gauge transformations to gauge away an infinite number of neutral bosonic ghost variables from the Lagrangian. The remaining components of the basic fields obey the Lagrangian field equations and these equations still posses necessary gauge invariance. Integrating the field equations back into a Lagrangian we complete the construction of gauge invariant Lagrangian and equations of motion in terms of a basic massive fermionic higher spin field and appropriate auxiliary fields interacting with a constant EM background.

Section 3 contains a more generic description in terms of so called "quartet formulation" [38,39] (see also [44]). This formulation is obtained from the one given in Section 2 by further use of the BRST gauge transformations to gauge away some auxiliary fields which are originally present in the system. In this way the Lagrangian contains only one physical field and six auxiliary fields three of which are Lagrangian multipliers. Let us note that in both cases the fields and the parameters of gauge transformations do not contain any off-shell conditions, unlike the formulation of [45].

In Section 4 we give a description of the simplest example of the spin $\frac{3}{2}$ field interacting with a constant EM background.

The final Section contains our conclusions and a discussion of some open problems.

2. Construction of gauge invariant Lagrangians

Let us briefly summarize the features of the BRST approach for the construction of the gauge invariant free and interacting Lagrangians (see [6] for a review). First one introduces a set of operators that define a spectrum of the theory.[3] Provided these operators form a closed algebra one builds a nilpotent BRST charge Q, which in turn yields to a quadratic gauge invariant Lagrangian of the form

$$\mathcal{L} \sim \langle\chi|Q|\chi\rangle \tag{2.1}$$

where $|\chi\rangle$ is a vector in an extended Fock space. The gauge invariance of the Lagrangian under the linear gauge transformations

$$\delta|\chi\rangle = Q|\Lambda\rangle \tag{2.2}$$

is guaranteed by the nilpotency of the BRST charge $Q^2 = 0$. This procedure is however slightly modified for the case of fermionic higher spin fields, since the condition of the BRST invariance

$$Q|\chi\rangle = 0 \tag{2.3}$$

cannot be integrated back into a Lagrangian in a straightforward way. Rather one uses a part of the gauge transformations (2.2) to gauge away a part of the auxiliary fields which are contained in $|\chi\rangle$. The resulting field equations turn out to be Lagrangian ones and they still possess enough gauge invariance to remove all nonphysical polarizations (see [19] for the details).

The situation is even more complicated if the closure of the algebra of the initial set of operators requires inclusion of certain additional operators into the system. These extra operators can impose too strong conditions on the field $|\chi\rangle$ so that there will be no nonzero solution to Eq. (2.3).

[3] In free theory the spectrum is given with the help of the relations defining either reducible or irreducible representations of the Poincare or AdS group.

A way out of this problem is the following (see [18–20] for fermionic higher spin fields and [6] for a detailed review of the BRST formulation for higher spin fields). One introduces additional sets of oscillator variables and builds auxiliary representation of the generators of the algebra (i.e. of the operators under consideration) in terms of these new variables. Then one defines a modified set of operators as a sum of new and old ones and therefore considers the problem in an extended Fock space. After that one builds BRST charge for modified generators in the standard way since the generators form a closed algebra. It allows us to construct a Lagrangian of the base of the BRST charge under consideration.

After this reminder let us turn to a description of massive fermionic higher spin fields. To this end we introduce the Fock space spanned by the oscillators

$$[a_\mu, a_\nu^+] = \eta_{\mu\nu}, \quad \eta_{\mu\nu} = diag(-1, 1, \dots, 1) \tag{2.4}$$

and consider the operators

$$t_0' = i\tilde{\gamma}^\mu \partial_\mu, \quad l_0' = \partial^2 - m^2, \quad l_1' = ia^\mu \partial_\mu, \quad t_1' = \tilde{\gamma}^\mu a_\mu, \quad l_2' = \tfrac{1}{2} a^\mu a_\mu. \tag{2.5}$$

Here we introduce Grassmann odd "gamma-matrix like objects" $\tilde{\gamma}^\mu$ and $\tilde{\gamma}$ which are connected with the usual Grassmann even gamma-matrices γ^μ by relation [18]

$$\gamma^\mu = \tilde{\gamma}^\mu \tilde{\gamma}, \quad \{\tilde{\gamma}^\mu, \tilde{\gamma}^\nu\} = -2\eta^{\mu\nu}, \quad \{\tilde{\gamma}^\mu, \tilde{\gamma}\} = 0, \quad \{\tilde{\gamma}, \tilde{\gamma}\} = -2. \tag{2.6}$$

The first of the operators in (2.5) corresponds to the Dirac operator for the massive fermion, the second operator is the d'Alembertian for a massive field, the third one is a divergence operator, the fourth one is an operator which takes a gamma-trace and the fifth one is an operator which takes a trace. In order to have a hermitian BRST charge we also introduce operators which are hermitian conjugate to the operators l_1', t_1' and l_2'

$$l_1'^{+} = ia^{+\mu}\partial_\mu, \quad t_1'^{+} = \tilde{\gamma}^\mu a_\mu^+, \quad l_2'^{+} = \tfrac{1}{2} a^{+\mu} a_\mu^+. \tag{2.7}$$

Finally in order to close the algebra one introduces the extra operators

$$g_0' = a^{+\mu} a_\mu + \frac{d}{2} \tag{2.8}$$

and $g_m' = m^2$. The operator g_0' is a "particle" number operator and its eigenvalues are always strictly positive. Therefore, we have a situation described earlier in this Section. We introduce three sets of additional oscillator variables: two sets of bosonic oscillator variables with commutation relations

$$[b_1, b_1^+] = 1, \quad [b_2, b_2^+] = 1, \tag{2.9}$$

and one set of fermionic oscillator variables

$$\{f, f^+\} = 1. \tag{2.10}$$

Using these new variables one can build auxiliary representation for the original operators and define modified operators as [18]

$$t_0 = i\tilde{\gamma}^\mu \partial_\mu - \tilde{\gamma} m \qquad l_0 = \partial^2 - m^2 \tag{2.11}$$

$$l_1 = ia^\mu \partial_\mu + m b_1 \qquad l_1^+ = ia^{+\mu}\partial_\mu + m b_1^+ \tag{2.12}$$

$$t_1 = \tilde{\gamma}^\mu a_\mu - \tilde{\gamma} b_1 + f^+ b_2 - 2(b_2^+ b_2 + h) f \qquad t_1^+ = a_\mu^+ \tilde{\gamma}^\mu - \tilde{\gamma} b_1^+ + f^+ - 2 b_2^+ f \tag{2.13}$$

$$l_2 = \tfrac{1}{2}a^\mu a_\mu + \tfrac{1}{2}b_1^2 + (b_2^+ b_2 + f^+ f + h)b_2 \qquad l_2^+ = \tfrac{1}{2}a^{+\mu}a_\mu^+ + \tfrac{1}{2}b_1^{+2} + b_2^+ \tag{2.14}$$

$$g_0 = a_\mu^+ a^\mu + b_1^+ b_1 + 2b_2^+ b_2 + f^+ f + \tfrac{d+1}{2} + h \qquad g_m = 0 \tag{2.15}$$

where h is an arbitrary real constant. The algebra of these operators is given by Table 1.

In order to introduce an interaction of the fermionic fields with an external constant EM background field $F_{\mu\nu} = const$ we shall proceed as follows. First we replace all the partial derivatives by the $U(1)$ covariant ones $D_\mu = \partial_\mu - ieA_\mu$ and include into the expressions of the operators[4] (2.11)–(2.15) terms which vanish in the limit $F_{\mu\nu} \to 0$. After that we require that the new operators form a closed algebra.

Before writing an ansatz for the operators let us note that since the trace of a field and its traceless part are independent from each other one can shift the trace of a field so that the traceless condition remains unchanged. Thus we suppose that the operators related with the traceless condition t_1, t_1^+, l_2, l_2^+ as well as the number operator g_0 remain unchanged

$$T_1 = t_1, \qquad T_1^+ = t_1^+, \qquad L_2 = l_2, \qquad L_2^+ = l_2^+ \qquad G_0 = g_0. \tag{2.16}$$

Moreover, since the oscillator variables b_2, b_2^+, f, f^+ (2.9)–(2.10) are included only in operators (2.16) (see also the expressions (2.13)–(2.15)) we assume that these variables are not present in the expressions of the operators T_0, L_0, L_1, L_1^+.

Since we are going to consider only the linear in $F_{\mu\nu}$ approximation we take the following ansatz for the operators

$$\begin{aligned} L_1 = {} & ia^\alpha D_\alpha + mb_1 + a^\alpha F_{\alpha\sigma} D^\sigma \sum_{k=0}^{\infty} f_{0k}\, b_1^{+k} b_1^k + \tilde{\gamma}\tilde{\gamma}^\tau F_{\tau\sigma} D^\sigma \sum_{k=0}^{\infty} f_{2k}\, b_1^{+k} b_1^{k+1} \\ & + a^{+\mu} F_{\mu\sigma} D^\sigma \sum_{k=0}^{\infty} f_{4k}\, b_1^{+k} b_1^{k+2} + \tilde{\gamma}^{\mu\nu} F_{\mu\nu} \sum_{k=0}^{\infty} d_{0k}\, b_1^{+k} b_1^{k+1} \\ & + \tilde{\gamma}\tilde{\gamma}^\sigma F_{\sigma\alpha} a^\alpha \sum_{k=0}^{\infty} d_{2k}\, b_1^{+k} b_1^k \\ & + a^{+\mu} F_{\mu\alpha} a^\alpha \sum_{k=0}^{\infty} d_{8k}\, b_1^{+k} b_1^{k+1} + \tilde{\gamma}\tilde{\gamma}^\sigma F_{\sigma\mu} a^{+\mu} \sum_{k=0}^{\infty} d_{4k}\, b_1^{+k} b_1^{k+2} \end{aligned} \tag{2.17}$$

$$\begin{aligned} T_0 = {} & i\tilde{\gamma}^\mu D_\mu - \tilde{\gamma} m + \tilde{\gamma}^\tau F_{\tau\sigma} D^\sigma \sum_{k=0}^{\infty} c_{0k}\, b_1^{+k} b_1^k \\ & + \tilde{\gamma} a^\alpha F_{\alpha\sigma} D^\sigma \sum_{k=0}^{\infty} c_{4k}\, (b_1^+)^{k+1} b_1^k + \tilde{\gamma} a^{+\mu} F_{\mu\sigma} D^\sigma \sum_{k=0}^{\infty} c_{5k}\, b_1^{+k} b_1^{k+1} \\ & + \tilde{\gamma}\tilde{\gamma}^{\mu\nu} F_{\mu\nu} \sum_{k=0}^{\infty} a_{0k}\, b_1^{+k} b_1^k + \tilde{\gamma} a^{+\mu} F_{\mu\alpha} a^\alpha \sum_{k=0}^{\infty} a_{4k}\, b_1^{+k} b_1^k \\ & + \tilde{\gamma}^\sigma F_{\sigma\alpha} a^\alpha \sum_{k=0}^{\infty} a_{2k}\, (b_1^+)^{k+1} b_1^k + \tilde{\gamma}^\sigma F_{\sigma\mu} a^{+\mu} \sum_{k=0}^{\infty} a_{3k}\, b_1^{+k} b_1^{k+1} \end{aligned} \tag{2.18}$$

[4] We shall denote these new operators by the corresponding capital letters.

Table 1
The algebra of the operators.

$[\downarrow,\rightarrow\}$	T_0	T_1	T_1^+	L_0	L_1	L_1^+	L_2	L_2^+	G_0
T_0	$2L_0$	$-2L_1$	$-2L_1^+$	0	0	0	0	0	0
T_1	$-2L_1$	$-4L_2$	$-2G_0$	0	0	T_0	0	T_1^+	T_1
T_1^+	$-2L_1^+$	$-2G_0$	$-4L_2^+$	0	$-T_0$	0	$-T_1$	0	$-T_1^+$
L_0	0	0	0	0	0	0	0	0	0
L_1	0	0	T_0	0	0	$-L_0$	0	L_1^+	L_1
L_1^+	0	$-T_0$	0	0	L_0	0	$-L_1$	0	$-L_1^+$
L_2	0	0	T_1	0	0	L_1	0	G_0	$2L_2$
L_2^+	0	$-T_1^+$	0	0	$-L_1^+$	0	$-G_0$	0	$-2L_2^+$
G_0	0	$-T_1$	T_1^+	0	$-L_1$	L_1^+	$-2L_2$	$2L_2^+$	0

$$
\begin{aligned}
L_1^+ &= ia^{+\mu}D_\mu + mb_1^+ + a^{+\mu}F_{\mu\sigma}D^\sigma\sum_{k=0}^{\infty} f_{1k}\, b_1^{+k}b_1^k + \tilde{\gamma}\tilde{\gamma}^\tau F_{\tau\sigma}D^\sigma \sum_{k=0}^{\infty} f_{3k}\,(b_1^+)^{k+1}b_1^k \\
&\quad + a^\alpha F_{\alpha\sigma}D^\sigma\sum_{k=0}^{\infty} f_{5k}\,(b_1^+)^{k+2}b_1^k + \tilde{\gamma}^{\mu\nu}F_{\mu\nu}\sum_{k=0}^{\infty} d_{1k}\,(b_1^+)^{k+1}b_1^k \\
&\quad + \tilde{\gamma}\tilde{\gamma}^\sigma F_{\sigma\mu}a^{+\mu}\sum_{k=0}^{\infty} d_{3k}\, b_1^{+k}b_1^k \\
&\quad + a^{+\mu}F_{\mu\alpha}a^\alpha\sum_{k=0}^{\infty} d_{9k}\,(b_1^+)^{k+1}b_1^k + \tilde{\gamma}\tilde{\gamma}^\sigma F_{\sigma\alpha}a^\alpha\sum_{k=0}^{\infty} d_{5k}\,(b_1^+)^{k+2}b_1^k
\end{aligned}
\tag{2.19}
$$

where $a_{ik}, c_{ik}, d_{ik}, c_{ik}$ are arbitrary complex constants and the rest of the operators (2.13)–(2.15) are unchanged as one can see form Eq. (2.16). Let us note that the above relations can be treated as the deformations of the corresponding relations of free theory by the terms linear in $F_{\mu\nu}$.

Let us point out that the ansatz for the operators L_1, T_0, L_1^+ (2.17)–(2.19) is not the most general one. The ansatz is taken on the basis of the following "minimal" rule. Let us consider the operators (2.17)–(2.19) in free theory, replace the partial derivatives by the covariant ones and calculate the commutators. Obviously the algebra will not be closed. Then one adds to these operators the minimal number of terms linear in $F_{\mu\nu}$ in such a way that the algebra is closed in the linear approximation. One can see that according to this "minimal" rule the Lorentz indices of the creation and annihilation operators are always contracted with the an index or indices of $F_{\mu\nu}$. In principle it is possible to consider other deformations of the free theory by the terms linear in $F_{\mu\nu}$. For example, one can add to L_1 a term of the form $a^\mu\gamma_\mu\gamma^\nu F_{\nu\sigma}D^\sigma$ but this term does not obey the "minimal" rule.

From the requirement the T_0 and L_0 to be hermitian, from the condition $(L_1)^+ = L_1^+$ and from the requirement that the total system of operators forms a closed algebra in the linear approximation one finds the expressions for constants which are presened in (2.17)–(2.19). These expressions are summarized in Appendix A.

Note that a similar problem was considered in [22], but we found two more arbitrary constants because, unlike [22], we do not require from the very beginning that the coefficients in (2.17)–(2.19) must satisfy reality conditions. As one can see from Appendix A, the complex coefficients are also acceptable.

The new operators form the algebra which is the same as in the free case and is given in Table 1.

After we have achieved the closure of the algebra for the operators, the next step is to construct the corresponding BRST charge. This procedure follows closely the one developed for the fermionic fields in [18,19] to which we refer for more details. First we construct the standard BRST operator on the basis of the operators (2.16)–(2.19)

$$\begin{aligned}
Q = {} & q_0 T_0 + q_1^+ T_1 + q_1 T_1^+ + \eta_0 L_0 + \eta_1^+ L_1 + \eta_1 L_1^+ + \eta_2^+ L_2 + \eta_2 L_2^+ + \eta_G G_0 \\
& + 2q_0(q_1^+ \mathcal{P}_1 + q_1 \mathcal{P}_1^+) + (q_1^+ \eta_1 - \eta_1^+ q_1) i p_0 + (\eta_1^+ \eta_1 - q_0^2)\mathcal{P}_0 + 2q_1^{+2}\mathcal{P}_2 \\
& + 2q_1^2 \mathcal{P}_2^+ + q_1^+ \eta_2 i p_1^+ - \eta_2^+ q_1 i p_1 - \eta_2^+ \eta_1 \mathcal{P}_1 - \eta_1^+ \eta_2 \mathcal{P}_1^+ + (2q_1^+ q_1 - \eta_2^+ \eta_2)\mathcal{P}_G \\
& + \eta_G(q_1^+ i p_1 - q_1 i p_1^+ + \eta_1^+ \mathcal{P}_1 - \eta_1 \mathcal{P}_1^+ + 2\eta_2^+ \mathcal{P}_2 - 2\eta_2 \mathcal{P}_2^+)
\end{aligned} \tag{2.20}$$

Here, q_0, q_1, q_1^+ and η_0, η_1^+, η_1, η_2^+, η_2, η_G are, respectively, the bosonic and fermionic ghost "coordinates" corresponding to their canonically conjugate ghost "momenta" p_0, p_1^+, p_1, $\mathcal{P}_0$, $\mathcal{P}_1$, $\mathcal{P}_1^+$, $\mathcal{P}_2$, $\mathcal{P}_2^+$, $\mathcal{P}_G$. They obey the (anti)commutation relations

$$\begin{aligned}
& \{\eta_1, \mathcal{P}_1^+\} = \{\mathcal{P}_1, \eta_1^+\} = \{\eta_2, \mathcal{P}_2^+\} = \{\mathcal{P}_2, \eta_2^+\} = \{\eta_0, \mathcal{P}_0\} = \{\eta_G, \mathcal{P}_G\} = 1, \\
& [q_0, p_0] = [q_1, p_1^+] = [q_1^+, p_1] = i
\end{aligned} \tag{2.21}$$

and possess the standard ghost number distribution, $gh(q,\eta) = -gh(p,\mathcal{P}) = 1$, which gives $gh(Q) = 1$.

For the subsequent computations it is convenient to present the BRST operator (2.20) in the form

$$\begin{aligned}
Q &= \tilde{Q} + \eta_G\big(N + \tfrac{d-3}{2} + h\big) + (2q_1^+ q_1 - \eta_2^+ \eta_2)\mathcal{P}_G \\
N &= a_\mu^+ a^\mu + b_1^+ b_1 + 2b_2^+ b_2 + f^+ f \\
&\quad + q_1^+ i p_1 - i p_1^+ q_1 + \eta_1^+ \mathcal{P}_1 + \mathcal{P}_1^+ \eta_1 + 2\eta_2^+ \mathcal{P}_2 + 2\mathcal{P}_2^+ \eta_2 \\
\tilde{Q} &= q_0 \tilde{T}_0 + \eta_0 L_0 + \Delta Q + (q_1^+ \eta_1 - \eta_1^+ q_1) i p_0 + (\eta_1^+ \eta_1 - q_0^2)\mathcal{P}_0 \\
\Delta Q &= q_1^+ T_1 + q_1 T_1^+ + \eta_1^+ L_1 + \eta_1 L_1^+ + \eta_2^+ L_2 + \eta_2 L_2^+ + 2q_1^{+2}\mathcal{P}_2 + 2q_1^2 \mathcal{P}_2^+ \\
&\quad + q_1^+ \eta_2 i p_1^+ - \eta_2^+ q_1 i p_1 - \eta_2^+ \eta_1 \mathcal{P}_1 - \eta_1^+ \eta_2 \mathcal{P}_1^+ \\
\tilde{T}_0 &= T_0 + 2q_1^+ \mathcal{P}_1 + 2q_1 \mathcal{P}_1^+.
\end{aligned}$$

Next we choose the following representation for the vacuum in the Hilbert space

$$(p_0, q_1, p_1, \mathcal{P}_0, \mathcal{P}_G, \eta_1, \mathcal{P}_1, \eta_2, \mathcal{P}_2)\,|0\rangle = 0\,, \tag{2.22}$$

and suppose that the vectors and gauge parameters do not depend on η_G,

$$\begin{aligned}
|\chi\rangle = {} & \sum_{k_i} (q_0)^{k_1}(q_1^+)^{k_2}(p_1^+)^{k_3}(\eta_0)^{k_4}(f^+)^{k_5}(\eta_1^+)^{k_6}(\mathcal{P}_1^+)^{k_7}(\eta_2^+)^{k_8}(\mathcal{P}_2^+)^{k_9}(b_1^+)^{k_{10}}(b_2^+)^{k_{11}} \\
& \times a^{+\mu_1}\cdots a^{+\mu_{k_0}} \chi^{k_1\cdots k_{11}}_{\mu_1\cdots\mu_{k_0}}(x)|0\rangle.
\end{aligned} \tag{2.23}$$

The sum in (2.23) is taken over k_0, k_1, k_2, k_3, k_{10}, k_{11}, running from 0 to infinity, and over $k_4, k_5, k_6, k_7, k_8, k_9$, running from 0 to 1. Then, we derive from Eqs. (2.3) as well as from the reducible gauge transformations, (2.2) a sequence of relations

$$\tilde{Q}|\chi\rangle = 0, \qquad (N + \tfrac{d-3}{2} + h)|\chi\rangle = 0, \qquad (\epsilon, gh)\,(|\chi\rangle) = (1, 0), \tag{2.24}$$
$$\delta|\chi\rangle = \tilde{Q}|\Lambda\rangle, \qquad (N + \tfrac{d-3}{2} + h)|\Lambda\rangle = 0, \qquad (\epsilon, gh)\,(|\Lambda\rangle) = (0, -1), \tag{2.25}$$
$$\delta|\Lambda\rangle = \tilde{Q}|\Lambda^{(1)}\rangle, \qquad (N + \tfrac{d-3}{2} + h)|\Lambda^{(1)}\rangle = 0, \qquad (\epsilon, gh)\,(|\Lambda^{(1)}\rangle) = (1, -2), \tag{2.26}$$
$$\delta|\Lambda^{(i-1)}\rangle = \tilde{Q}|\Lambda^{(i)}\rangle, \qquad (N + \tfrac{d-3}{2} + h)|\Lambda^{(i)}\rangle = 0, \qquad (\epsilon, gh)\,(|\Lambda^{(i)}\rangle) = (i, -i-1). \tag{2.27}$$

Here ϵ defines a Grassmann parity of corresponding fields and parameters of gauge transformations as $(-1)^{\epsilon}$.

The middle equation in (2.24) is a constraint on possible values of h

$$h = 2 - s - \frac{d}{2}. \tag{2.28}$$

By fixing the value of spin, we also fix the parameter h, according to (2.28). Having fixed a value of h, we then substitute it into each of the expressions (2.24)–(2.27).

Analogously to the free case [18] the equation of motion (2.24) cannot be obtained from a Lagrangian. In order to extract from (2.24) a Lagrangian set of equations of motion we decompose the state vector and gauge parameters in terms of powers of neutral Grassmann even q_0, and Grassmann odd η_0 ghosts

$$|\chi\rangle = \sum_{k=0}^{\infty} q_0^k(|\chi_0^k\rangle + \eta_0|\chi_1^k\rangle), \qquad |\Lambda\rangle = \sum_{k=0}^{\infty} q_0^k(|\Lambda_0^k\rangle + \eta_0|\Lambda_1^k\rangle).$$

Then we remove all fields except $|\chi_0^0\rangle$ and $|\chi_0^1\rangle$ using a part of the initial gauge symmetries or using their own equations of motion. As a result of this procedure Eq. (2.24) is reduced to

$$\Delta Q|\chi_0^0\rangle + \tfrac{1}{2}\{\tilde{T}_0, \eta_1^+\eta_1\}|\chi_0^1\rangle = 0, \quad \tilde{T}_0|\chi_0^0\rangle + \Delta Q|\chi_0^1\rangle = 0. \tag{2.29}$$

These equations are invariant under the gauge transformations

$$\delta|\chi_0^0\rangle = \Delta Q|\Lambda_0^0\rangle + \tfrac{1}{2}\{\tilde{T}_0, \eta_1^+\eta_1\}|\Lambda_0^1\rangle \quad \delta|\chi_0^1\rangle = \tilde{T}_0|\Lambda_0^0\rangle + \Delta Q|\Lambda_0^1\rangle. \tag{2.30}$$

The parameters of gauge transformations are in turn invariant under the chain of transformations with a finite number of reducibility stages $i_{\max} = s - 3/2$

$$\delta|\Lambda^{(i)}{}_0^0\rangle = \Delta Q|\Lambda^{(i+1)}{}_0^0\rangle + \frac{1}{2}\big\{\tilde{T}_0, \eta_1^+\eta_1\big\}|\Lambda^{(i+1)}{}_0^1\rangle, \quad |\Lambda^{(0)}{}_0^0\rangle_n = |\Lambda_0^0\rangle, \tag{2.31}$$
$$\delta|\Lambda^{(i)}{}_0^1\rangle = \tilde{T}_0|\Lambda^{(i+1)}{}_0^0\rangle + \Delta Q|\Lambda^{(i+1)}{}_0^1\rangle, \qquad |\Lambda^{(0)}{}_0^1\rangle_n = |\Lambda_0^1\rangle, \tag{2.32}$$
$$i_{\max} = s - 3/2 \tag{2.33}$$

where $\{\tilde{T}_0, \eta_1^+\eta_1\} = \tilde{T}_0\eta_1^+\eta_1 + \eta_1^+\eta_1\tilde{T}_0$.

It is straightforward to check that Eqs. (2.29) can be obtained from the following Lagrangian

$$\mathcal{L} = \langle\tilde{\chi}_0^0|K_h\Big\{\tilde{T}_0|\chi_0^0\rangle + \Delta Q|\chi_0^1\rangle\Big\} + \langle\tilde{\chi}_0^1|K_h\Big\{\Delta Q|\chi_0^0\rangle + \tfrac{1}{2}\{\tilde{T}_0, \eta_1^+\eta_1\}|\chi_0^1\rangle\Big\}. \tag{2.34}$$

In (2.34) operator K_h

$$K_h = \sum_{n=0}^{\infty}\frac{1}{n!}\Big(|n\rangle\langle n|C(n,h) - 2f^+|n\rangle\langle n|f\,C(n+1,h)\Big),$$
$$C(n,h) = h(h+1)\cdots(h+n-1), \qquad C(0,h) = 1, \qquad |n\rangle = (b_2^+)^n|0\rangle \tag{2.35}$$

is needed to maintain hermiticity of the Lagrangian since as one can see from the auxiliary representations for operators (2.13)–(2.14) one has $(l_2)^+ \neq l_2^+$ and $(t_1)^+ \neq t_1^+$. The fields $\langle\tilde{\chi}_0^0|$, $\langle\tilde{\chi}_0^1|$ are defined as follows

$$\langle\tilde{\chi}_0^0| = (|\chi_0^0\rangle)^+\tilde{\gamma}^0, \qquad \langle\tilde{\chi}_0^1| = (|\chi_0^1\rangle)^+\tilde{\gamma}^0. \tag{2.36}$$

The Lagrangian (2.34) describes the interaction of massive fermionic fields with constant electromagnetic field and it is our main result. It contains, apart from the basic field $\psi_{\mu_1\ldots\mu_n}(x)$ in $|\chi_0^0\rangle$

$$|\chi_0^0\rangle = \psi_{\mu_1\ldots\mu_n}(x)a^{+\mu_1}\ldots a^{+\mu_n}|0\rangle + \ldots \tag{2.37}$$

a number of auxiliary fields,[5] whose number increases with spin value.

Let us make some comments. The first comment is about causal propagation. If one has a system of the first order differential equations for a set of fields φ^B

$$G_B^{A\mu}\partial_\mu\varphi^B + \ldots = 0, \qquad \mu, \nu = 0, \ldots, d-1 \tag{2.38}$$

then, following [40,41], in order to verify that the system (2.38) describes hyperbolic propagation one should check that all solutions $n_0(n_i)$, $(i = 1, \ldots d-1)$ of the algebraic equation

$$\det(G_B^{A\mu}n_\mu) = 0 \tag{2.39}$$

are real for any given real set of n_μ. The hyperbolic system is called causal if there are no timelike vectors among solutions n_μ of (2.39). In the free case one has $\det(G_B^{A\mu}n_\mu) = (n^2)^C = 0$, where value of C depends on the dimension of the space–time and on the spin of the field φ^B. Therefore propagation of the field is hyperbolic and causal. On the other hand if one considers propagation of a field in constant electromagnetic background then one has

$$\det(G_B^{A\mu}n_\mu) = (n^2)^C + O(F^2) = 0. \tag{2.40}$$

Eq. (2.40) does not have a contribution which is linear in $F_{\mu\nu}$ due to antisymmetry of $F_{\mu\nu}$ and symmetry of a product of any number of n_μ. Due to this fact propagation of fields in a constant electromagnetic field is hyperbolic and causal in the linear approximation in $F_{\mu\nu}$.

The second comment concerns the number of physical degrees of freedom. The obtained Lagrangian (2.34) is a deformation of the free Lagrangian [18] (see also [20] for fermionic fields in AdS). Therefore, the interacting theory under consideration contains the same number of physical degrees of freedom as the corresponding free theory. In [18] it was shown that the obtained Lagrangian and gauge transformations indeed reproduce the equations on the basic field which define irreducible representation of the Poincare (or AdS) group. Since after the switching on of the interactions the number of the fields and the symmetries is the same as in the free case, the number of the physical degrees of freedom is preserved. Let us note that a set of auxiliary fields for free massive fermionic higher spin field theory has been discovered in [52] on the basis of the requirement that the equations of motion must identically reproduce the conditions on a basic field determining the irreducible representation of the Poincare group. We would like to emphasize that the BRST approach is based on the same requirement: the equations of motion for the vector in the Fock space obtained in the BRST approach must identically reproduce the conditions on a basic field which determine an irreducible representation of the Poincare group

[5] In decomposition (2.23) there are the coefficients in summands which contain at least one creation operator different from $a^{+\mu}$.

in terms of Fock space vectors. Since both the BRST approach and the Singh–Hagen formulation are based on the same basic requirement, it is clear that the sets of auxiliary fields in both formulations are related to each other. On the other hand since the BRST approach is more generic, the Singh–Hagen formulation can be obtained from the BRST approach as a very special partial case.[6] However, the BRST approach allows one to derive many interesting Lagrangian formulations which are equivalent to each other on-shell. These formulations include different sets of auxiliary fields which can be used in different contexts. For example, there is a possibility to derive in an universal way a gauge invariant formulation for higher spin fields which contains Stueckelberg fields; the formulation for higher spin fields in terms of triplets [46–50]; or the formulation in terms of quartets [38,39]; as well as a formulation with reducible gauge transformations which can be interesting from the point of view of general gauge theory.

3. Lagrangian formulations with a smaller number of auxiliary fields

In this Section we are going to obtain from (2.34) different Lagrangian formulations partially fixing the gauge invariance.

First we derive a quartet Lagrangian formulation [38,39]. Initially this formulation was developed for the massless higher spin fields in flat and AdS background in [38]. Its fermionic version contains seven unconstrained fields (one physical field and six auxiliary fields three of which are Lagrangian multipliers) and one unconstrained gauge parameter.[7] Using dimensional reduction one can obtain the quartet formulation for massive higher spin fields in Minkowski space [39].

To obtain this formulation from the Lagrangian (2.34) we partially fix gauge invariance just as it was done in [20], except we will not fix gauge invariance corresponding to gauge parameter $|\varepsilon\rangle$

$$|\Lambda^{(0)0}{}_0\rangle = \mathcal{P}_1^+|\varepsilon\rangle + \ldots, \qquad |\varepsilon\rangle = \sum_{k=0}^{n-1}\frac{1}{k!}(b_1^+)^k|\varepsilon_{n-k-1}\rangle \tag{3.1}$$

$$|\varepsilon_{n-k-1}\rangle = \frac{1}{(n-k)!}\, a^{+\mu_1}\ldots a^{+\mu_{n-k-1}}\varepsilon_{\mu_1\ldots\mu_{n-k-1}}(x)|0\rangle. \tag{3.2}$$

Next one can show that after the gauge fixing some of the remaining fields can be removed with the help of the equations of motion and the nonvanishing fields in the quartet formulation are

$$|\chi_0^0\rangle = |\Psi^{(n)}\rangle + \eta_1^+\mathcal{P}_1^+|D^{(n-2)}\rangle + q_1^+\mathcal{P}_1^+|E^{(n-2)}\rangle + i\eta_1^+p_1^+|\Sigma^{(n-2)}\rangle \tag{3.3}$$

$$|\chi_0^1\rangle = \mathcal{P}_1^+|C^{(n-1)}\rangle - ip_1^+|\Lambda^{(n-1)}\rangle + ip_1^+\eta_1^+\mathcal{P}_1^+|\Omega^{(n-3)}\rangle. \tag{3.4}$$

The Lagrangian and the gauge transformation for the massive fermionic higher spin field interacting with constant electromagnetic field in the quartet formulation are[8]

$$\begin{aligned}\mathcal{L} = {}& \langle\tilde{\Psi}^{(n)}|\Big\{T_0|\Psi^{(n)}\rangle + L_1^+|C^{(n-1)}\rangle + T_1'^+|\Lambda^{(n-1)}\rangle\Big\} \\ & - \langle\tilde{C}^{(n-1)}|\Big\{T_0|C^{(n-1)}\rangle - L_1|\Psi^{(n)}\rangle + L_1^+|D^{(n-2)}\rangle - |\Lambda^{(n-1)}\rangle - T_1'^+|\Sigma^{(n-2)}\rangle\Big\}\end{aligned}$$

---

[6] A possibility to derive the Singh–Hagen formulation from the BRST approach is discussed in Section 3.

[7] Another similar formulation (so-called triplet formulation) of fermionic fields on Minkowski and AdS_d backgrounds contains one physical and two auxiliary fields [46–50] (see also [51] for a recent discussion) and corresponds to a description of reducible representations of the Poincare or $SO(d-2,2)$ groups.

[8] In order to obtain triplet formulation [46–50] one should to discard field $|E^{(n-2)}\rangle$ and Lagrangian multipliers $|\Lambda^{(n-1)}\rangle$, $|\Sigma^{(n-2)}\rangle$, $|\Omega^{(n-3)}\rangle$ in (3.5).

$$
\begin{aligned}
&- \langle\tilde{D}^{(n-2)}|\Big\{T_0|D^{(n-2)}\rangle + L_1|C^{(n-1)}\rangle + 2|\Sigma^{(n-2)}\rangle - T_1'^{+}|\Omega^{(n-3)}\rangle\Big\}\\
&+ \langle\tilde{\Lambda}^{(n-1)}|\Big\{T_1'|\Psi^{(n)}\rangle + |C^{(n-1)}\rangle + L_1^{+}|E^{(n-2)}\rangle\Big\}\\
&+ \langle\tilde{\Sigma}^{(n-2)}|\Big\{T_1'|C^{(n-1)}\rangle - 2|D^{(n-2)}\rangle + T_0|E^{(n-2)}\rangle\Big\}\\
&+ \langle\tilde{\Omega}^{(n-3)}|\Big\{T_1'|D^{(n-2)}\rangle + L_1|E^{(n-2)}\rangle\Big\}\\
&+ \langle\tilde{E}^{(n-2)}|\Big\{L_1|\Lambda^{(n-1)}\rangle + T_0|\Sigma^{(n-2)}\rangle + L_1^{+}|\Omega^{(n-3)}\rangle\Big\}
\end{aligned} \tag{3.5}
$$

$$
\delta|\Psi^{(n)}\rangle = L_1^{+}|\Upsilon^{(n-1)}\rangle, \qquad \delta|C^{(n-1)}\rangle = -T_0|\Upsilon^{(n-1)}\rangle, \tag{3.6}
$$

$$
\delta|D^{(n-2)}\rangle = L_1|\Upsilon^{(n-1)}\rangle, \qquad \delta|E^{(n-2)}\rangle = -T_1'|\Upsilon^{(n-1)}\rangle. \tag{3.7}
$$

The fields and the gauge parameter $|\Upsilon^{(n-1)}\rangle \equiv |\varepsilon^{(n-1)}\rangle$ depend only on the oscillators (a_μ, b_1). In particular in (3.5)–(3.7) the fields and the gauge parameter have uniform decomposition

$$
|\Phi^{(m)}\rangle = \sum_{k=0}^{m} \frac{1}{k!}(b_1^{+})^k|\phi_{m-k}\rangle \tag{3.8}
$$

$$
|\phi_{m-k}\rangle = \frac{1}{(n-k)!}\, a^{+\mu_1}\dots a^{+\mu_{m-k}}\phi_{\mu_1\dots\mu_{m-k}}(x)|0\rangle \tag{3.9}
$$

and the operators T_1' and $T_1'^{+}$ are the (a_μ, b_1) parts of the operators t_1 and t_1^{+} (2.13)

$$
T_1' = \tilde{\gamma}^\mu a_\mu - \tilde{\gamma} b_1 \qquad T_1'^{+} = a_\mu^{+}\tilde{\gamma}^\mu - \tilde{\gamma} b_1^{+}. \tag{3.10}
$$

Next we will show that the Lagrangian formulation, which obtained in [22], is a particular case of our general result (3.5). To get such Lagrangian formulations we first partly fix the gauge, removing the field $|E^{(n-2)}\rangle$ with the help of gauge transformations (3.7) and then integrate out all the fields except the field $|\Psi^{(n)}\rangle$. The result is

$$
\begin{aligned}
\mathcal{L} = \langle\tilde{\Psi}^{(n)}|\Big\{&T_0 - L_1^{+}T_1' - T_1'^{+}L_1 - T_1'^{+}T_0T_1'\\
&- \tfrac{1}{2}T_1'^{+}L_1^{+}T_1'T_1' - \tfrac{1}{2}T_1'^{+}T_1'^{+}L_1T_1' - \tfrac{1}{4}T_1'^{+}T_1'^{+}T_0T_1'T_1'\Big\}|\Psi^{(n)}\rangle
\end{aligned} \tag{3.11}
$$

$$
\delta|\Psi^{(n)}\rangle = L_1^{+}|\Upsilon^{(n-1)}\rangle \tag{3.12}
$$

where the state $|\Psi^{(n)}\rangle$ and the parameter of gauge transformations $|\Upsilon^{(n-1)}\rangle$ obey the constraints

$$
(T_1')^3|\Psi^{(n)}\rangle = 0, \qquad T_1'|\Upsilon^{(n-1)}\rangle = 0. \tag{3.13}
$$

Such a partial form of the Lagrangian was obtained in [22], but with another (less general[9]) expressions for the operators (2.16)–(2.19).

We can proceed to obtain more Lagrangian formulations. For example, we can resolve constraints on the field and the gauge parameter (3.13). Using decomposition (3.9) for $|\Psi^{(n)}\rangle$ and $|\Upsilon^{(n-1)}\rangle$

[9] It should be noted that in [22] was considered deformation of the operators corresponding to the gamma-traceless conditions as well. But this deformation is proportional to an arbitrary constant and as we said at the beginning of our paper can be removed by a field redefinition.

$$|\Psi^{(n)}\rangle = \sum_{k=0}^{n} \frac{1}{k!}(b_1^+)^k|\psi_{n-k}\rangle \qquad |\Upsilon^{(n-1)}\rangle = \sum_{k=0}^{n-1} \frac{1}{k!}(b_1^+)^k|\epsilon_{n-1-k}\rangle \tag{3.14}$$

we find that gauge parameter $|\epsilon_{n-1}\rangle$ is not restricted and the other parameters $|\epsilon_k\rangle$ are expressed in terms of its gamma-traces $|\epsilon_k\rangle = (\gamma^\mu a_\mu)^{n-1-k}|\epsilon_{n-1}\rangle$, so we may make gauge transformation using the unrestricted gauge parameter $|\epsilon_{n-1}\rangle$. One can do the same for the field $|\Psi^{(n)}\rangle$. Due to restriction (3.13) there are only three independent fields $|\psi_n\rangle$, $|\psi_{n-1}\rangle$, $|\psi_{n-2}\rangle$ and all the other fields are expressed through these three fields

$$|\psi_{n-2k-1}\rangle = -k(\gamma^\mu a_\mu)^{2k+1}|\psi_n\rangle + (\gamma^\mu a_\mu)^{2k}|\psi_{n-1}\rangle + (k+1)(\gamma^\mu a_\mu)^{2k-1}|\psi_{n-2}\rangle \tag{3.15}$$

$k \geq 1$,

$$|\psi_{n-2k-2}\rangle = -k(\gamma^\mu a_\mu)^{2k+2}|\psi_n\rangle + (k+1)(\gamma^\mu a_\mu)^{2k}|\psi_{n-2}\rangle. \tag{3.16}$$

Thus one can obtain[10] a gauge invariant Lagrangian formulation for a massive fermionic field interacting with constant electromagnetic field with the help of three fields $|\psi_n\rangle$, $|\psi_{n-1}\rangle$, $|\psi_{n-2}\rangle$ and one gauge parameter $|\epsilon_{n-1}\rangle$.

Finally, using the remaining unrestricted gauge parameter $|\epsilon_{n-1}\rangle$ one can remove the field $|\psi_{n-1}\rangle$ and obtain a Lagrangian formulation in terms of two γ-tracefull unrestricted fields: a basic field $|\psi_n\rangle$, which after the total gauge fixing and using the equations of motion satisfies the conditions of irreducible representation of the Poincare group, and an additional auxiliary field $|\psi_{n-2}\rangle$. This Lagrangian has no gauge invariance since we have already used the entire gauge freedom. If one further decomposes the γ-tracefull fields $|\psi_n\rangle$ and $|\psi_{n-2}\rangle$ into a sum of γ-traceless fields one obtains a set of the fields which exactly coincides with the one given in the Singh–Hagen formulation [52].

4. Example: spin 3/2

In this section we apply a general procedure described in the previous Sections for the simplest example of spin-3/2 field.

In the case of spin-3/2 field we have $h = \frac{1-d}{2}$ (see Eq. (2.28)) and since according to (2.33) we have $i_{max} = 0$. Therefore the corresponding Lagrangian formulation is an irreducible gauge theory. Due to $gh(|\Lambda_0^1\rangle_1 = -2)$, the nonvanishing fields $|\chi_0^0\rangle_1$, $|\chi_0^1\rangle_1$ and the gauge parameter $|\Lambda_0^0\rangle_1$ (we have $|\Lambda_0^1\rangle_1 \equiv 0$), possess the following Grassmann grading and ghost number distributions:

$$(\varepsilon, gh)\,(|\chi_0^0\rangle_1) = (1,0), \quad (\varepsilon, gh)\,(|\chi_0^1\rangle_1) = (1,-1), \quad (\varepsilon, gh)\,(|\Lambda_0^0\rangle_1) = (0,-1). \tag{4.1}$$

These conditions determine the dependence of the fields and of the gauge parameters on the oscillator variables in a unique form

$$\begin{aligned}
|\chi_0^0\rangle_1 &= \big[ia^{+\mu}\psi_\mu(x) + f^+\tilde{\gamma}\psi(x) + b_1^+\varphi(x)\big]|0\rangle, \\
\langle\tilde{\chi}_0^0| &= \langle 0|\big[-\psi_\mu^+(x)ia^\mu + \psi^+(x)\tilde{\gamma}f + \varphi^+(x)b_1\big]\tilde{\gamma}^0, \\
|\chi_0^1\rangle_1 &= \big[\mathcal{P}_1^+\tilde{\gamma}\chi(x) - ip_1^+\chi_1(x)\big]|0\rangle, \\
\langle\tilde{\chi}_0^1| &= \langle 0|\big[\chi_1^+(x)ip_1 + \chi^+(x)\tilde{\gamma}\mathcal{P}_1\big]\tilde{\gamma}^0, \\
|\Lambda_0^0\rangle_1 &= \big[\mathcal{P}_1^+\lambda(x) - ip_1^+\tilde{\gamma}\lambda_1(x)\big]|0\rangle \qquad h = -\tfrac{d-1}{2}.
\end{aligned}$$

[10] Since the Lagrangian formulation is very large, we do not present it here.

Substituting these expressions for the fields and the gauge parameters in (2.34) and (2.30) one finds the Lagrangian and gauge transformations for the physical spin-3/2 field ψ^μ and for the auxiliary fields

$$\begin{aligned}
\mathcal{L}_{3/2} = \bar{\psi}^\mu \Big[& (i\gamma^\sigma D_\sigma - m)\psi_\mu + \frac{ie}{2m^2}(2\zeta_1 + \xi_1)\gamma^\tau F_{\tau\sigma} D^\sigma \psi_\mu - \frac{ie}{8m}(1+4\zeta_0)\gamma^{\alpha\beta}F_{\alpha\beta}\psi_\mu \\
& + \frac{e}{m^2}(\xi_1 - 2i\zeta_0)F^{\mu\sigma}D_\sigma\varphi - \frac{e}{2m}(1-2i\xi_1)\gamma^\tau F_{\tau\mu}\varphi - \frac{ie}{2m}(1-4\zeta_0)F_{\mu\nu}\psi^\nu \\
& + D_\mu\chi + \frac{e}{m^2}(\zeta_1 + i\zeta_0)F_{\mu\sigma}D^\sigma\chi - \frac{e}{4m}(1+2i\xi_1)\gamma^\tau F_{\tau\mu}\chi - i\gamma_\mu\chi_1 \Big] \\
& + \bar{\varphi}\Big[(i\gamma^\mu D_\mu - m)\varphi + \frac{ie}{2m^2}(2\zeta_1 - 3\xi_1)\gamma^\tau F_{\tau\sigma}D^\sigma\varphi - \frac{e}{m^2}(\xi_1 + 2i\zeta_0)F^{\mu\sigma}D_\sigma\psi_\mu \\
& - \frac{e}{2m}(1+2i\xi_1)\gamma_\tau F^{\tau\mu}\psi_\mu - \frac{ie}{8m}(1+4\zeta_0)\gamma^{\mu\nu}F_{\mu\nu}\varphi \\
& + m\chi + \frac{e}{2m^2}(2\zeta_0 + i\xi_1)\gamma^\tau F_{\tau\sigma}D^\sigma\chi - \frac{ie}{8m}(1-4\zeta_0+4i\xi_1)\gamma^{\mu\nu}F_{\mu\nu}\chi - \chi_1\Big] \\
& - (d-1)\bar{\psi}\Big[(i\gamma^\mu D_\mu + m)\psi + \frac{ie}{2m^2}(2\zeta_1+\xi_1)\gamma^\tau F_{\tau\sigma}D^\sigma\psi \\
& + \frac{ie}{8m}(1+4\zeta_0)\gamma^{\mu\nu}F_{\mu\nu}\psi - \chi_1\Big] \\
& - \bar{\chi}\Big[\Big\{i\gamma^\mu D_\mu + m + \frac{ie}{2m^2}(2\zeta_1+\xi_1)\gamma^\tau F_{\tau\sigma}D^\sigma + \frac{ie}{8m}(1+4\zeta_0)\gamma^{\mu\nu}F_{\mu\nu}\Big\}\chi - \chi_1 \\
& + \Big\{D_\mu + \frac{e}{m^2}(\zeta_1 - i\zeta_0)F_{\mu\sigma}D^\sigma + \frac{e}{4m}(1-2i\xi_1)\gamma^\tau F_{\tau\mu}\Big\}\psi^\mu \\
& - m\varphi + \frac{e}{2m^2}(2\zeta_0 - i\xi_1)\gamma^\tau F_{\tau\sigma}D^\sigma\varphi + \frac{ie}{8m}(1-4\zeta_0-4i\xi_1)\gamma^{\mu\nu}F_{\mu\nu}\varphi\Big] \\
& + \bar{\chi}_1\Big[\chi + i\gamma^\mu\psi_\mu - \varphi + (d-1)\psi\Big]
\end{aligned} \tag{4.2}$$

$$\begin{aligned}
\delta\psi_\mu &= \Big\{D_\mu + \frac{e}{m^2}(\zeta_1 + i\zeta_0)F_{\mu\sigma}D^\sigma + \frac{e}{4m}(1+2i\xi_1)\gamma^\sigma F_{\sigma\mu}\Big\}\lambda - i\gamma_\mu\lambda_1, \\
\delta\psi &= \lambda_1, \\
\delta\varphi &= \Big\{m - \frac{e}{2m^2}\big(2\zeta_0 + i\xi_1\big)\gamma^\tau F_{\tau\sigma}D^\sigma - \frac{ie}{8m}(1-4\zeta_0+4i\xi_1)\gamma^{\mu\nu}F_{\mu\nu}\Big\}\lambda + \lambda_1, \\
\delta\chi &= \Big\{-i\gamma^\mu D_\mu + m - \frac{ie}{2m^2}(2\zeta_1+\xi_1)\gamma^\tau F_{\tau\sigma}D^\sigma + \frac{ie}{8m}(1+4\zeta_0)\gamma^{\mu\nu}F_{\mu\nu}\Big\}\lambda + 2\lambda_1, \\
\delta\chi_1 &= \Big\{i\gamma^\mu D_\mu + m + \frac{ie}{2m^2}(2\zeta_1+\xi_1)\gamma^\tau F_{\tau\sigma}D^\sigma + \frac{ie}{8m}(1+4\zeta_0)\gamma^{\mu\nu}F_{\mu\nu}\Big\}\lambda_1.
\end{aligned} \tag{4.3}$$

Here we have used that $K_h f^+|0\rangle = -2hf^+|0\rangle$ with substitution $-2h \to (d-1)$.

Thus we have derived from the general Lagrangian the one which contains component fields and the corresponding gauge transformations. This Lagrangian describes a massive field with spin 3/2, coupled to a constant electromagnetic background in the linear approximation and contains a number of free parameters.[11] The relations (4.2)–(4.3) are our final results. One can

[11] The problem of Lagrangian formulation for spin-3/2 field coupled to EM field in a linear approximation has been studied in [35] where the Lagrangian also contains a number of free parameters. However, unlike our paper, it is has

further eliminate the auxiliary fields using the gauge freedom and some of the equations of motion and thus obtain the field equations for only basic field ψ^{μ}.

Let us make some comments. First, when constructing interacting theories there is a possibility to generate so called "fake interactions" i.e., the ones which can be obtained from a free Lagrangian via field redefinitions. The terms which describe these kind of interactions vanish when one formally uses the free equations of motion. It is easy to see that none of the terms which describes the interaction of the field ψ^{μ} with a constant electromagnetic background in Eq. (4.2) is of this type, and therefore all parameters which are present in the Lagrangian are arbitrary from this point of view. Second, the Lagrangian (4.2) was derived in the framework of the gauge invariant approach. It contains a minimal number of arbitrary free parameters which is compatible with the gauge invariance.[12]

5. Conclusions

In the present paper we have developed the BRST approach to construct and analyze a Lagrangian description of massive higher spin fermionic fields interacting with constant electromagnetic field in the linear approximation. To this end, we modified the operators underlying the BRST charge which corresponds to the noninteracting fermionic massive higher spin fields by terms depending on the electromagnetic field. The obtained Lagrangian contains apart from the basic field also some number auxiliary fields which provide the gauge invariant description for massive theory, and the number of these fields grows with the value of the spin.

We also showed that one can partially or completely fix the gauge invariance and obtain a family of different Lagrangian formulations with a smaller number of auxiliary fields. As an example we derived a Lagrangian formulation for the massive fermionic higher spin fields interacting with a constant electromagnetic background in the quartet formulation [38,39] and obtained the results of paper [22] as a particular case. Also we gave a detailed description of the component Lagrangian and gauge transformations for a simplest example of the spin $\frac{3}{2}$ field interacting with a constant electromagnetic background.

Since in the present paper we have considered fermionic higher spin fields it would be naturally interesting to generalize the present results for the case of supersymmetric systems[13] as well as to consider higher order interactions. Inclusion of a nontrivial gravitational background is yet another interesting problem to consider (see for example [54–56] for recent progress in these directions). It would be interesting also to establish more connection with the recent studies in conformal higher spin fields (see for example [57–61]). We hope to address these questions in future publications.

been assumed in [35] that the electromagnetic field is dynamical and moreover, the model under consideration possesses a certain amount of supersymmetries. These requirements impose the some strong restrictions on the structure of the Lagrangian. As a result, the Lagrangian (4.2) contains more free parameters in comparison with the Lagrangian given in [35].

12 We would like to emphasize that it is not possible to reduce a number of arbitrary parameters in the off-shell Lagrangian (4.2) using the field redefinitions without partial or total violation of gauge invariance. The purpose of our paper is however to develop a gauge invariant formulation.

13 Lagrangian formulation of free supersymmetric massive higher spin theory was done in [53].

Acknowledgements

I.L.B. is grateful to the grant for Leading Russian Scientific Schools, project 88.2014.2 and RFBR grant, project No. 15-02-03594 for partial support. Research of V.A.K. was supported by Russian Ministry of Education and Science under contract No. 3.867.2014/K. The work of M.T. has been supported in part by an Australian Research Council grant DP120101340. M.T. would also like to acknowledge grant 31/89 of the Shota Rustaveli National Science Foundation.

Appendix A. Expressions for free parameters

Below we give the expressions for free parameters which are present in Eqs. (2.17)–(2.19)

$$
\begin{aligned}
a_{0(0)} &= -\frac{ie}{8m} - \frac{ie}{2m}\zeta_0 & a_{0(k)} &= 0 \quad k \geq 1 \\
a_{2(0)} &= \frac{ie}{2m} - \frac{e}{m}\xi_1 & a_{2(k)} &= -\frac{(-2)^k}{k!}\frac{e}{m}\xi_1 \quad k \geq 1 \\
a_{3(0)} &= -\frac{ie}{2m} - \frac{e}{m}\xi_1 & a_{3(k)} &= -\frac{(-2)^k}{k!}\frac{e}{m}\xi_1 \quad k \geq 1 \\
a_{4(0)} &= -\frac{ie}{2m} + \frac{2ie}{m}\zeta_0 & a_{4(k)} &= 0 \quad k \geq 1
\end{aligned}
$$

$$
\begin{aligned}
c_{0(0)} &= \frac{ie}{2m^2}(2\zeta_1 + \xi_1) & c_{0(k)} &= \frac{(-2)^k}{k!}\frac{ie}{m^2}\xi_1 \quad k \geq 1 \\
c_{4(0)} &= \frac{e}{m^2}(-2\zeta_0 + i\xi_1) & c_{4(k)} &= \frac{(-2)^k}{k!}\frac{ie}{m^2}\xi_1 \quad k \geq 1 \\
c_{5(0)} &= \frac{e}{m^2}(2\zeta_0 + i\xi_1) & c_{5(k)} &= \frac{(-2)^k}{k!}\frac{ie}{m^2}\xi_1 \quad k \geq 1
\end{aligned}
$$

$$
\begin{aligned}
d_{0(0)} &= -\frac{ie}{8m} + \frac{ie}{2m}(\zeta_0 + i\xi_1) & d_{0(k)} &= \frac{(-2)^{k-1}}{k!}\frac{e}{m}\xi_1 \quad k \geq 1 \\
d_{1(0)} &= -\frac{ie}{8m} + \frac{ie}{2m}(\zeta_0 - i\xi_1) & d_{1(k)} &= -\frac{(-2)^{k-1}}{k!}\frac{e}{m}\xi_1 \quad k \geq 1 \\
d_{2(0)} &= -\frac{ie}{4m} - \frac{e}{2m}\xi_1 & d_{2(k)} &= \frac{(-2)^{k-1}}{k!}(k+1)\frac{e}{m}\xi_1 \quad k \geq 1 \\
d_{3(0)} &= -\frac{ie}{4m} + \frac{e}{2m}\xi_1 & d_{3(k)} &= -\frac{(-2)^{k-1}}{k!}(k+1)\frac{e}{m}\xi_1 \quad k \geq 1
\end{aligned}
$$

$$
\begin{aligned}
d_{4(k)} &= \frac{(-2)^k}{k!}\frac{e}{m}\xi_1 \quad k \geq 0 \\
d_{5(k)} &= -\frac{(-2)^k}{k!}\frac{e}{m}\xi_1 \quad k \geq 0
\end{aligned}
$$

$$
\begin{aligned}
d_{8(0)} &= \frac{ie}{m} - \frac{ie}{m}(2\zeta_0 + i\xi_1) & d_{8(k)} &= \frac{(-2)^k}{k!}\frac{e}{m}\xi_1 \quad k \geq 1 \\
d_{9(0)} &= \frac{ie}{m} - \frac{ie}{m}(2\zeta_0 - i\xi_1) & d_{9(k)} &= -\frac{(-2)^k}{k!}\frac{e}{m}\xi_1 \quad k \geq 1
\end{aligned}
$$

$$f_{0(0)} = \frac{e}{m^2}(\zeta_0 + i\zeta_1)$$
$$f_{0(1)} = \frac{e}{m^2}(2\zeta_0 - i\xi_1) \qquad f_{0(k)} = -\frac{(-2)^{k-1}}{k!}\frac{ie}{m^2}\xi_1 \quad k \geq 2$$
$$f_{1(0)} = \frac{e}{m^2}(-\zeta_0 + i\zeta_1)$$
$$f_{1(1)} = -\frac{e}{m^2}(2\zeta_0 + i\xi_1) \qquad f_{1(k)} = -\frac{(-2)^{k-1}}{k!}\frac{ie}{m^2}\xi_1 \quad k \geq 2$$
$$f_{2(0)} = \frac{e}{m^2}(\zeta_0 - \frac{i}{2}\xi_1) \qquad f_{2(k)} = \frac{(-2)^{k-1}}{k!}\frac{ie}{m^2}\xi_1 \quad k \geq 1$$
$$f_{3(0)} = \frac{e}{m^2}(\zeta_0 + \frac{i}{2}\xi_1) \qquad f_{3(k)} = -\frac{(-2)^{k-1}}{k!}\frac{ie}{m^2}\xi_1 \quad k \geq 1$$
$$f_{4(0)} = -\frac{2e}{m^2}\zeta_0 \qquad f_{4(k)} = 0 \quad k \geq 1$$
$$f_{5(0)} = \frac{2e}{m^2}\zeta_0 \qquad f_{5(k)} = 0 \quad k \geq 1.$$

Here ζ_0, ζ_1, ξ_1 are arbitrary real dimensionless constants.

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